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Stability Analysis of Distributed Autonomous Vehicles

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Abstract: This paper describes how large-scale decentralized control theory may be used to analyze the stability of multiple cooperative robotic vehicles. Models of cooperation are discussed from a decentralized control system point of view. Whereas decentralized control research in the past has concentrated on using decentralized controllers to partition complex physically interconnected systems, this work uses decentralized methods to connect otherwise independent non-touching robotic vehicles so that they behave in a stable, coordinated fashion. A vector Liapunov method is used to prove stability of two examples: the controlled motion of multiple vehicles along a perimeter and the controlled motion of multiple vehicles in a plane.

Keywords: Mobile robotics, Lyapunov-based control, decentralized control, swarms.

1. INTRODUCTION

Researchers have recently begun to investigate decentralized control techniques to control multiple autonomous vehicles. Chen and Luh [1] examined decentralized control laws that drove a set of mobile robots into a circle formation. Similarly, Yamaguchi studied line-formations [2] and general formations [3], and so did Yoshida et al, [4]. Decentralized control laws using a potential field approach to guide vehicles away from obstacles can be found in [5-6]. Beni and Liang [7] prove the convergence of a linear swarm of distributed autonomous vehicles into a synchronously achievable configuration. Work by Liu and Passino [12] addresses stability of an asynchronous swarm in which the vehicles have proximity sensors and use information from nearest neighbors for positioning.

In this paper, we address the stable control of multiple vehicles using large-scale decentralized control techniques [8]. Our goal here is to show how a simple control law for each vehicle can guarantee stability of the overall swarm. In prior work, we described how to test for controllability and observability of a large-scale system [9] as well as some simulation and hardware tests on a ground-based swarm [11,13]. The goal of this work is to coordinate the behavior of a large number (10s to 100s to 1000s) of autonomous robotic vehicles performing various tasks such as reconnaissance, surveillance, hazardous environmental operations, physical security, and logistics support.

Once we know that a system is structurally observable and controllable, the next question to ask is that of connective stability. Will the overall system be globally asymptotically stable under structural perturbations? Analysis of connective stability is based upon the concept of vector Liapunov functions, which associates several scalar functions with a dynamic system in such a way that each function guarantees stability in different portions of the state space. The objective is to prove that there exist Liapunov functions for each of the individual subsystems and then prove that the vector sum of these Liapunov functions is a Liapunov function for the entire system.

2. STABILITY OF LARGE SCALE SYSTEMS

Suppose that the overall system is denoted by

$$\mathbf{S}: \quad \begin{aligned} \dot{x} &= f(t,x,u) \\ y &= h(t,x) \end{aligned} \tag{1}$$

where $x(t)\in \Re^n$ is the state of $\mathbf{S}$ (e.g., x, y position, orientation, and linear and angular velocities of all vehicles) at time $t \in T$, $u(t)\in \Re^m$ are the inputs (e.g., the commanded wheel velocities of all vehicles), and $y(t)\in \Re^\ell$ are the outputs (e.g., GPS measured x,y position of all vehicles). The function $f: T\times \Re^n \times \Re^m \to \Re^n$ describes the dynamics of $\mathbf{S}$, and the function $h: T\times \Re^n \to \Re^\ell$ describes the observations of $\mathbf{S}$. We can partition the system into N interconnected subsystems given by

$$\mathbf{S}: \quad \dot{x}_i = f_i(t, x_i, u_i) + \widetilde{f}_i(t, x, u), i \in \{1, \ldots, N\}$$
$$y_i = h_i(t, x_i) + \widetilde{h}_i(t, x) \qquad (2)$$

where $x_i(t) \in \Re^{n_i}$ is the state of the ith subsystem $\mathbf{S}_i$ at time $t \in \Re$, $u_i(t) \in \Re^{m_i}$ are the inputs to $\mathbf{S}_i$, and $y_i(t) \in \Re^{\ell_i}$ are the outputs of $\mathbf{S}_i$. The function $f_i : T \times \Re^{n_i} \times \Re^{m_i} \rightarrow \Re^{n_i}$ describes the dynamics of $\mathbf{S}_i$, and the function $\widetilde{f}_i : T \times \Re^{n} \times \Re^{m} \rightarrow \Re^{n_i}$ represents the dynamic interaction of $\mathbf{S}_i$ with the rest of the system $\mathbf{S}$. The function $h_i : T \times \Re^{n_i} \rightarrow \Re^{\ell_i}$ represents observations at $\mathbf{S}_i$ derived only from local state variables of $\mathbf{S}_i$, and the function $\widetilde{h}_i : T \times \Re^{n} \rightarrow \Re^{\ell_i}$ represents observation at $\mathbf{S}_i$ derived from the rest of $\mathbf{S}$. The N independent subsystems are denoted as

$$\mathbf{S}_i: \quad \dot{x}_i = f_i(t, x_i, u_i), \qquad i \in \{1, \ldots, N\}$$
$$y_i = h_i(t, x_i) \qquad (3)$$

Both local and interconnected feedback may be added to the system with

$$u_i = k_i(t, y_i) + \widetilde{k}_i(t, y), \qquad i \in \{1, \ldots, N\} \qquad (4)$$

where the function $k_i : T \times \Re^{\ell_i} \rightarrow \Re^{m_i}$ represents the feedback at $\mathbf{S}_i$ derived only from local observations, and the function $\widetilde{k}_i : T \times \Re^{\ell} \rightarrow \Re^{m_i}$ represents the feedback at $\mathbf{S}_i$ derived from the rest of $\mathbf{S}$. For stability analysis, we will assume that the control function has already been chosen and the closed loop dynamics of the system can be written as

$$\mathbf{S}: \quad \dot{x}_i = g_i(t, x_i) + \widetilde{g}_i(t, x), \qquad i \in \{1, \ldots, N\}. \qquad (5)$$

where the function $g_i : T \times \Re^{n_i} \rightarrow \Re^{n_i}$ describes the closed loop dynamics of $\mathbf{S}_i$. The closed loop interconnection function can be written as

$$\widetilde{g}_i(t, x) = \widetilde{g}_i(t, \overline{e}_{i1}x_1, \overline{e}_{i2}x_2, \ldots, \overline{e}_{iN}x_N), \quad i \in \{1, \ldots, N\} \qquad (6)$$

where $\overline{e}_{ij} \in B^{n_i \times n_j}$, and the elements of the fundamental interconnection matrix $\overline{E} = (\overline{e}_{ij})$ are

$$(\overline{e}_{ij})_{pq} = \begin{cases} 1, & (x_j)_q \text{ occurs in } (\widetilde{g}_i(t, x, u))_p \\ 0, & (x_j)_q \text{ does not occur in } (\widetilde{g}_i(t, x, u))_p. \end{cases}$$

(7)

where $q \in \{n_j\}$ and $p \in \{n_i\}$.

The structural perturbations of $\mathbf{S}$ are introduced by assuming that the elements of the fundamental interconnection matrix that are one can be replaced by any number between zero and one, i.e.

$$e_{ij} = \begin{cases} [0,1], & \overline{e}_{ij} = 1 \\ 0, & \overline{e}_{ij} = 0. \end{cases} \qquad (8)$$

Therefore, the elements e_{ij} represent the strength of coupling between the individual subsystems. A system is connectively stable if it is stable in the sense of Liapunov for all possible $E = (e_{ij})$ [8]. In other words, if a system is connectively stable, it is stable even if an interconnection becomes decoupled, i.e. $e_{ij} = 0$, or if interconnection parameters are perturbed, i.e. $0 < e_{ij} < 1$. This is potentially very powerful, as it proves that the system will be stable if an interconnection is lost through communication failure.

For a possibly nonlinear system $\mathbf{S}$ to be connectively stable, there must exist a matrix $W = (w_{ij})$ that is an M-matrix (i.e. all leading principal minors must be positive):

$$w_{ij} = \begin{cases} 1 - \overline{e}_{ii}\kappa_i\xi_{ii}, & i = j \\ -\overline{e}_{ij}\kappa_i\xi_{ij}, & i \neq j \end{cases} \qquad (9)$$

where $\kappa_i > 0$, and the scalar function $v_i : T \times \Re^{n_i} \rightarrow \Re_+$ must satisfy a Lipschitz condition

$$\left| v_i(t, x') - v_i(t, x_i'') \right| \le \kappa_i \left\| x_i' - x_i'' \right\|, \quad \forall t \in T, \quad \forall x_i', x_i'' \in \Re^{n_i} \qquad (10)$$

Also, the constant $\xi_{ij} \ge 0$ for $i \neq j$ and satisfy

$$\left\| \widetilde{g}_i(t, x) \right\| \le \sum_{j=1}^{N} \overline{e}_{ij}\xi_{ij}\phi_j\left(\left\| x_j \right\|\right), \quad \forall (t, x) \in T \times \Re^{n} \qquad (11)$$

where the time derivative of the Liapunov function is less than the negative of the comparison function $\phi_j\left(\left\| x_j \right\|\right)$

$$\dot{v}_i(t, x_i) \le -\phi_j\left(\left\| x_j \right\|\right), \quad \forall (t, x_i) \in T \times \Re^{n_i}. \qquad (12)$$

For linear systems, the matrix W is a function of the eigenvalues of the state transition matrix. Suppose the linear system dynamics are

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + \sum_{j=1}^{N} e_{ij} A_{ij} x_j, \qquad i \in \{1, \ldots, N\}, \qquad (13)$$

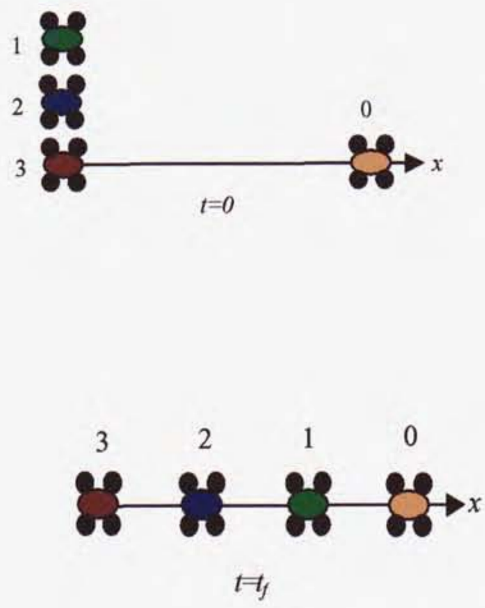
and the Liapunov function for each individual subsystem is $v_i(x_i) = \left(x_i^T H_i x_i\right)^{1/2}$ where H_i is a positive definite matrix. For the system $\mathbf{S}$ to be connectively stable, the following test matrix $W = (w_{ij})$ must be an M-matrix [8]:

$$
w_{ij} = \begin{cases} \dfrac{\lambda_m(G_i)}{2\lambda_M(H_i)} - \bar{e}_{ii}\lambda_M^{1/2}\left(A_{ii}^T A_{ii}\right) & i = j \\ -\bar{e}_{ij}\lambda_M^{1/2}\left(A_{ij}^T A_{ij}\right) & i \neq j \end{cases} \tag{14}
$$

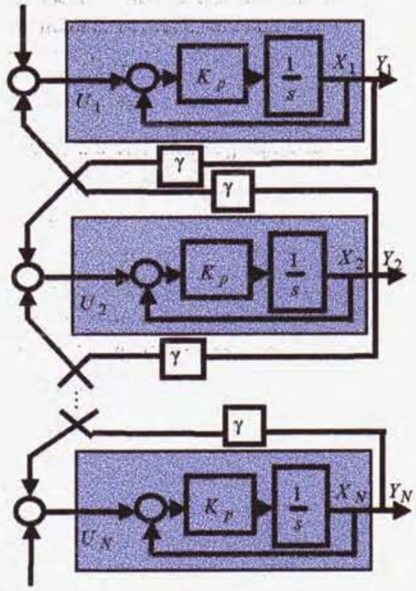
where the symmetric positive definite matrix G_i satisfies the Liapunov matrix equation $A_i^T H_i + H_i A_i = -G_i$, and $\lambda_m(\bullet)$ and $\lambda_M(\bullet)$ are the minimum and maximum eigenvalues of the corresponding matrices. This same analysis can also be performed in the discrete domain [10].

2.1 Example of a Linear Interconnected System

As an example, let us analyze a simple lin ear one-dimensional problem in which a chain of interdependent vehicles is to spread out along a line as shown in Figure 1(a). The objective is to spread out evenly along the line using only information from the nearest neighbor.



(a)



(b)

Figure 1. (a) One-dimensional control problem. The top line is the initial state. The second line is the desired final state. Vehicle s 0 and 3 are boundary conditions. Vehicles 1 and 2 spread out along the line by using only the position of their left and right neighbor. (b) Control block diagram of N -vehicle interaction problem.

Assume that the vehicle's plant is modeled as a simple integrator, and the commanded input is the desired velocity of the vehicle along the line. A feedback loop and a proportional gain K_p are used to control each vehicle's position (see Figure 1(b)). The dynamics of each subsystem is

$$
\mathbf{S}_i : \begin{aligned} \dot{x}_i &= -K_p x_i + K_p u_i, \qquad i \in \{1,\dots,N\} \\ y_i &= x_i \end{aligned} \tag{15}
$$

where x_i is the position of the ith vehicle, u_i is the control input, and y_i is the observation. Assume the control of each vehicle is a function of the two nearest vehicles' observed positions, and the boundary conditions on the first and last vehicle are 1 and 0, respectively.

$$
\begin{aligned} u_1 &= 1 + \gamma y_2 \\ u_i &= \gamma\left(y_{i-1} + y_{i+1}\right) \qquad i \in \{2,\dots,N-1\} \\ u_N &= \gamma y_{N-1} \end{aligned} \tag{16}
$$

where γ is the interaction gain between vehicles. For this linear system, the test matrix becomes

$$W=\begin{bmatrix} K_p & -K_p\gamma & 0 & \cdots & 0 \\ -K_p\gamma & K_p & -K_p\gamma & & \vdots \\ 0 & -K_p\gamma & K_p & & 0 \\ \vdots & & & \ddots & -K_p\gamma \\ 0 & \cdots & 0 & -K_p\gamma & K_p \end{bmatrix}$$

(17)

For $N=2$, this test matrix is an M-matrix (i.e. the system is connectively stable) if $|\gamma|<1$. For $N=3$, the system is connectively stable if $|\gamma|<\frac{1}{\sqrt{2}}$. For $N=4$, the system is connectively stable if $|\gamma|<0.618$. Notice how the range of the interaction gain gets smaller for larger sized systems. In fact, for this particular example, the interaction gain range reaches a limit of $|\gamma|\le 0.5$ for infinite numbers of vehicles. More details on the linear interconnected case can be found in [11] including discussion of the stability "house" that describes the stability boundaries of the system w.r.t. γ and K_p that can be derived from the M-matrix in (17).

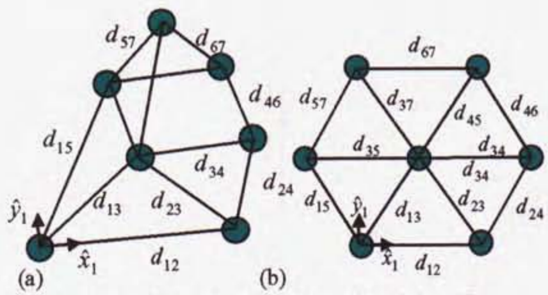


Figure 2. (a) Initial configuration of vehicles. (b) Desired configuration.

2.2 Example of a Non-Linear Interconnected System

Next, let us consider the problem of N vehicles spreading out in a two-dimensional space while staying a specified distance from their neighbors (See Figure 2). We assume that the vehicles communicate their position to their neighbors and that each vehicle knows the distance that it is suppose to be from neighboring vehicles. Is there a decentralized control that will drive the group of vehicles to the desired configuration?

To solve such a problem, a gradient-based control law is proposed and a vector Liapunov technique [8] is used to prove stability. The dynamics of the vehicles are essentially ignored so that the vehicle dynamics can be considered to be the control law only.

$$\dot{\bar{x}}_i=\bar{u}_i,\quad i=1,\dots,N \tag{18}$$

where $\bar{x}_i=[x_i\, y_i]^T\in\Re^2$ is the ith vehicle state space vector, and $\bar{u}\in\Re^2$ is the control input. The scalar values x_i and y_i are the x and y coordinates of the ith vehicle. A Liapunov function is defined for each vehicle that is minimized when the vehicle is a specified distance from the other vehicles.

$$v_i(t,\bar{x}_i)=\sum_{j=1}^{N} e_{ij}\left[d_{ij}^2-(x_i-x_j)^2-(y_i-y_j)^2\right]^2$$

(19)

where

$$e_{ij}=\begin{cases}0, & j\text{ is not nearest neighbor}\\ 1, & j\text{ is nearest neighbor}\end{cases} \tag{20}$$

and d_{ij} are the desired distances between the i and j vehicles (note $d_{ij}=d_{ji}$ and $d_{ii}=0$). The decentralized Liapunov functions v_i are a measure of the sum of the squared errors in distance for vehicle i with respect to all the neighboring vehicles. Since this function is not zero at $\bar{x}_i=0$, a new state vector $\ddot{x}_i=\bar{x}_i-\bar{x}_{i0}$ is defined such that

$$d_{ij}^2=\left\|\bar{x}_{i0}-\bar{x}_{j0}\right\|^2 \tag{21}$$

where $\bar{x}_{i0}$ is the final position of the ith vehicle after the vehicles are dispersed and is considered a constant. Then the Liapunov function for the ith vehicle can be written as

$$v_i(t,\ddot{x}_i)=\sum_{j=1}^{N} e_{ij}\left[\left\|\ddot{x}_i-\ddot{x}_j\right\|^2-2(\ddot{x}_i-\ddot{x}_j)^T(\bar{x}_{i0}-\bar{x}_{j0})\right]^2 \tag{22}$$

which equals zero when $\ddot{x}_i=0$ and is greater than zero for $\ddot{x}_i\neq 0$.

In order to minimize the ith vehicle Liapunov function, we use a control law that is the spatial gradient of the Liapunov function:

$$\dot{\bar{x}}_i=\bar{u}_i=-\alpha\frac{\partial v_i(t,\bar{x}_i)}{\partial\bar{x}_i},i=1,\dots,N \tag{23}$$

where $\alpha>0$ is the control gain. The time derivative of the ith vehicle Liapunov function is given by

$$\dot{v}_i(t,\bar{x}_i)=\frac{\partial v_i(t,\bar{x}_i)}{\partial t}+\frac{\partial v_i(t,\bar{x}_i)}{\partial\bar{x}_i}\dot{\bar{x}}_i=-\alpha\left\|\frac{\partial v_i(t,\bar{x}_i)}{\partial\bar{x}_i}\right\|^2$$

(24)

where

$$\frac{\partial v_i(t,\bar{x}_i)}{\partial \bar{x}_i} = -4\sum_{j=1}^{N} e_{ij}\left(d_{ij}^2 - \left\|\bar{x}_i - \bar{x}_j\right\|^2\right)(\bar{x}_i - \bar{x}_j)$$

$$= -4\sum_{j=1}^{N} e_{ij}\left[-\left\|\bar{x}_i - \bar{x}_j\right\|^2 - 2(\ddot{x}_i - \ddot{x}_j)^T(\bar{x}_{i0} - \bar{x}_{j0})\right] \tag{25}$$

$$\bullet\left[\left(\ddot{x}_i - \ddot{x}_j\right) + \left(\bar{x}_{i0} - \bar{x}_{j0}\right)\right]$$

Since $\dot{v}_i \le 0$ and it is equal to zero only at $\vec{x}_i = 0$, this is a valid Liapunov function and the gradient-based control law is stable for a single vehicle.

The next step is to show that when all vehicles use the same control law that the entire system is stable. We assume that the Liapunov function for the entire system can be described as a vector Liapunov function (the sum of the individual Liapunov functions)

$$v(t,\vec{x}) = \sum_{j=1}^{N} \rho_j v_j(t,\vec{x}_j) \tag{26}$$

where $\rho_j > 0$. Clearly, $v \ge 0$ for all $\vec{x} \in \Re^n$ and it is equal to zero only if $\vec{x} = 0$. We want to show that $\dot{v}(t,\vec{x}) \le 0$ for all $\vec{x} \in \Re^n$ and it is equal to zero only if $\vec{x} = 0$. The time derivative of the vector Liapunov function is

$$\dot{v}(t,\vec{x}) = \frac{\partial v(t,\vec{x})}{\partial t} + \left[\frac{\partial v(t,\vec{x})}{\partial \vec{x}}\right]^T \dot{\vec{x}} \tag{27}$$

Since $v(t,\vec{x})$ is independent of time, the first term on the right is zero. If $\rho_i = 1$ and $e_{ij} = e_{ji}$ for all $i, j \in N$, then the second term is

$$\dot{v}(t,\vec{x}) = -16\alpha \sum_{i=1}^{N} \eta_i \tag{28}$$

where

$$\eta_i = z_i^T X_i^T X_i z_i \tag{29}$$

$$X_i = \left[(\bar{x}_i - \bar{x}_1) \quad \cdots \quad (\bar{x}_i - \bar{x}_N)\right] \in \Re^{2\times N} \tag{30}$$

$$z_i = \left[e_{i1}\left(d_{i1}^2 - \left\|\bar{x}_i - \bar{x}_1\right\|^2\right) \cdots e_{iN}\left(d_{iN}^2 - \left\|\bar{x}_i - \bar{x}_N\right\|^2\right)\right]^T \in \Re^{N\times 1} \tag{31}$$

Since $X_i^T X_i$ is a positive semi-definite matrix, then $\eta_i \ge 0$ for $i \in \{1,\dots,N\}$ and $\dot{v}(t,\vec{x}) \le 0$ for all $\vec{x} \in \Re^n$. The elements of this semi-definite matrix are

$$\left(X_i^T X_i\right)_{pq} = \left\|\bar{x}_i - \bar{x}_p\right\|\left\|\bar{x}_i - \bar{x}_q\right\|\cos\theta_{piq} \tag{32}$$

where θ_{piq} is the angle between vectors $\bar{x}_i - \bar{x}_p$ and $\bar{x}_i - \bar{x}_p$. The derivative of the vector Liapunov is equal to zero only when $\vec{x} = 0$, which is the same as $d_{ij}^2 - \left\|\bar{x}_i - \bar{x}_j\right\|^2 = 0$ or $z_i = 0$ for all $i, j \in \{1,\dots,N\}$. This proves that the system Liapunov function $v(t,\vec{x})$ is valid, and the decentralized gradient-based control law drives the entire system to a stable configuration.

3. CONCLUSIONS

In this paper, we mathematically described how to determine if a cooperative robotic system is connectively stable. We illustrated the use of this technique on both a linear and a non-linear problem. The control law for the linear problem has been applied to robotic perimeter surveillance task. The control law for the non-linear problem has been applied to a building surveillance task. Hardware implementation of these control algorithms is being conducted on the vehicles depicted in Fig. 3. In addition, high fidelity simulations using a modeling and simulation tool at Sandia called Umbra [14] are also ongoing as shown in Fig. 4 (details can be found in [11]).

4. REFERENCES

[1] Chen, Qin and Luh, J.Y.S., "Coordination and Control of a Group of Small Mobile Robots," IEEE International Conference on Robotics and Automation, Vol. 3, 1994, pp. 2315-2320.

[2] H. Yamaguchi, and T. Arai, "Distributed and Autonomous Control Method for Generating Shape of Multiple Mobile Robot Group," *Proceedings of the IEEE International Conference on Intelligent Robots and Systems,* Vol. 2, 1994, pp.800-807.

[3] H. Yamaguchi, J.W. Burdick, "Asymptotic Stabilization of Multiple Nonholonomic Mobile Robots Forming Group Formations," *Proceedings of the 1998 Conference on Robotics & Automation*, Leuven, Belgium, May 1998, 3573-3580.

[4] E. Yoshida, T. Arai, J. Ota, and T. Miki, "Effect of Grouping in Local Communication System of Multiple Mobile Robots," *Proceedings of the IEEE International Conference on Intelligent Robots and Systems,* Vol. 2, 1994, pp. 808-815.

[5] P. Molnar and J. Starke, "Communication Fault Tolerance in Distributed Robotic Systems," *Distributed Autonomous Robotic Systems 4*, ed. L.E. Parker, G. Bekey, J. Barhen, Springer-Verlag 2000, pp. 99-108.

[6] F. E. Schneider, D. Wildermuth, H. -L. Wolf, "Motion Coordination in Formations of Multiple Robots Using a Potential Field Approach," *Distributed Autonomous Robotic Systems 4*, ed. L.E. Parker, G. Bekey, J. Barhen, Springer-Verlag 2000, pp. 305-314.

[7] G. Beni and P. Liang, "Pattern Reconfiguration in Swarms – Convergence of a Distributed Asynchronous and Bounded Iterative Algorithm," *IEEE Transactions on Robotics and Automation*, Vol. 12, No. 3, June 1996, pp. 485-490.
[8] D. D. Siljak, *Decentralized Control of Complex Systems*, Academic Press, 1991.
[9] J.T. Feddema, D.A. Schoenwald, "Decentralized Control of Cooperative Robotic Systems," *Proceedings of SPIE,* Vol. 4364, AeroSense, Orlando, Florida, April 16, 2001.
[10] M.E. Sezer and D.D. Siljak, "Robust Stability of Discrete Systems," *Int. J. Control*, Vol. 48, No. 5, pp. 2055-2063, 1988.
[11] D.A. Schoenwald, J. T. Feddema, F.J. Oppel, "Decentralized Control of a Collective of Autonomous Robotic Vehicles," *Proceedings of the American Control Conference*, Arlington, VA, June 25-27, 2001.
[12] Y. Liu, K. Passino, and M. Polycarpou, "Stability Analysis of One-Dimensional Asynchronous Swarms," 2001 American Control Conference, Arlington, VA, June 25-27, 2001.
[13] J. Feddema, C. Lewis, and D. Schoenwald, "Decentralized Control of Cooperative Robotic Vehicles: Theory and Application," to appear in October 2002 issue of *IEEE Transactions on Robotics and Automation.*
[14] E. J. Gottlieb, R. W. Harrigan, M. J. McDonald, F. J. Oppel, and P. G. Xavier, "The Umbra Simulation Framework," Sandia National Laboratories Report, SAND2001-1533, June 2001.

Figure 3. Experimental testing of robotic vehicles setting up a communication/navigation network inside a building.

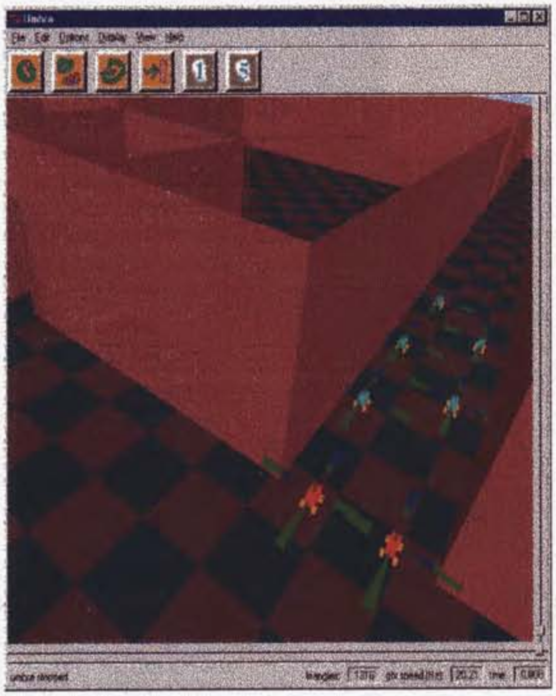


Figure 4. Screen shot of an Umbra high fidelity simulation of a collective of robotic vehicles navigating a building.

5. ACKNOWLEDGEMENTS

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